Dealing with $(a + b)^n$

Remember $\quad (a + b)^n = a^n \left(1 + \dfrac{b}{a}\right)^n$

Examples

1. Find first four terms in the binomial expansion of $\sqrt{4 + x}$. State the values of x for which the expansion is valid.

Quickfire First Step

What would be the first step in finding the Binomial expansion of each of these?

	First Step…	Valid when?
1. $(2 + x)^{-3}$		
2. $(9 + 2x)^{\frac{1}{2}}$		
3. $(8 - x)^{\frac{1}{3}}$		
4. $(5 - 2x)^{-3}$		
5. $(16 + 3x)^{-\frac{1}{2}}$		

Test Your Understanding

(a) Find the binomial expansion of

$$\sqrt{(9 + 8x)}, \quad |x| < \frac{9}{8}$$

in ascending powers of x, up to and including the term in x^2.
Give each coefficient as a simplified fraction.

(5)

(b) Use your expansion to estimate the value of $\sqrt{(11)}$, giving your answer as a single fraction.

(3)

Extension

[AEA 2006 Q1]

(a) For $|y| < 1$, write down the binomial series expansion of $(1 - y)^{-2}$ in ascending powers of y up to and including the term in y^3.

(b) Hence, or otherwise, show that
$$1 + \frac{2x}{1 + x} + \frac{3x^2}{(1 + x)^2} + \cdots + \frac{rx^{r-1}}{(1 + x)^{r-1}} + \cdots$$
can be written in the form $(a + x)^n$. Write down the values of the integers a and n.

(c) Find the set of values of x for which the series in part (b) is convergent.

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